Phys 234H Final Exam Study Guide (Chapters 42-43)

The exam will be specifically connected to the learning outcomes for the course, listed on the course web page, and repeated below:

1. Recall the definitions and relationships involving oscillations and waves, such as wavelength, frequency, angular frequency, amplitude, phase, wave speed, restoring force, longitudinal and transverse waves, standing waves, damping, interference, diffraction, doppler shift, and other wave phenomena.
2. Comprehend the meaning of the equations governing oscillations and waves, and be able to manipulate them to obtain any desired quantitative relationship. Understand the extension of these equations to the quantum realm (wave-particle duality) for free particles, atoms and nuclei. Generalize the concepts underlying the equations, such as restoring force, inertia, energy.
3. Apply the equations governing oscillations and waves to mechanical systems for various boundary conditions, to optical systems, and to quantum physics in atomic and nuclear systems. Calculate unknown quantities based on physical relationships, boundary conditions, and known quantities.
4. Analyze graphs of oscillatory and wave phenomena to obtain wavelength, frequency, amplitude, phase, particle and wave position, velocity, acceleration, damping time constant, as a function of time. Indentify and distinguish types of wave motion such as transverse, longitudinal, standing waves, reflection, refraction.
5. Evaluate the soundness and precision of your answers. Explain and interpret your solutions to problems in a way that shows deeper understanding. Identify and appraise the range of applicability of your results, and their limitations.
6. Devise your own small experiment to demonstrate some phenomenon in one of the categories of geometric optics, interference, or diffraction.

Outcome 1: The important definitions we have met in the new chapters include:

$E = \dfrac{L^2}{2I}$ (relation between E and L);

$L = \sqrt{l(l+1)}\,\hbar$ (quantized molecular rotational angular momentum—quantum number $l = 0, 1, 2, \ldots$);

$E_l = l(l+1)\dfrac{\hbar^2}{2I}$ (rotational energy levels for diatomic molecule);

$\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ (reduced mass of a diatomic molecule with masses m_1 and m_2);

$I = \mu r_0^2$ (moment of inertia for diatomic molecule with reduced mass μ and separation r_0);

$E_n = (n+\tfrac{1}{2})\hbar\omega = (n+\tfrac{1}{2})\hbar\sqrt{\dfrac{k'}{\mu}}$ (vibrational energies for molecule—quantum number $n = 0, 1, 2, \ldots$);

$\Delta n = \pm 1;\ \Delta l = \pm 1$ (quantum-mechanical rules for allowed transitions);

$g(E) = \frac{(2m)^{3/2} V}{2\pi^2 \hbar^3} E^{1/2}$ (density of states, free-electron model); $f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$ (Fermi-Dirac distribution, where E_F is the Fermi energy); $E_{\mathrm{F0}} = \frac{3^{2/3} \pi^{4/3} \hbar^2}{2m} \left(\frac{N}{V}\right)^{2/3}$ (Fermi energy at absolute zero—N/V is electron concentration); $E_{\mathrm{av}} = \frac{3}{5} E_{\mathrm{F0}}$ (average free-electron energy); $dN = g(E) f(E) dE$ (number of electrons in energy range between E and $E+dE$);

$I = I_S (e^{eV/kT} - 1)$ (current through a p-n junction, where I_S is the limiting reverse current)

$R = R_0 A^{1/3}$ (radius of nucleus of given A); $R_0 = 1.2 \times 10^{-15}$ m $= 1.2$ fm (const. of proportionality);

$A = Z + N$ (relation between atomic mass number, charge, and neutron number);

$S = \sqrt{s(s+1)}\,\hbar = \sqrt{\frac{3}{4}}\,\hbar$ (quantized nuclear spin angular momentum—quantum number $s = 1/2$);

$S_z = m_s \hbar$ (quantized z-component of spin angular momentum—quantum number $m_s = \pm\frac{1}{2}$);

$J = \sqrt{j(j+1)}\,\hbar$ (quantized total nuclear angular momentum—quantum number $j = 0, 1, 2, \ldots$);

$J_z = m_j \hbar$ (quantized z-component of total ang. momentum—quantum number $m_j = 0, \pm 1, \pm 2, \ldots \pm j$);

$U_{\mathrm{proton}} = \pm 2.7928 \mu_n B$ (interaction energy of proton spin angular momentum in B field);

$U_{\mathrm{neutron}} = \pm 1.9130 \mu_n B$ (interaction energy of proton spin angular momentum in B field);

$\mu_n = \frac{e\hbar}{2m_p} = 3.15245 \times 10^{-8}$ eV/T (Nuclear magneton);

$E_B = (ZM_H + Nm_n - {}^A_Z M)c^2$ (nuclear binding energy); c^2 = 931.5 MeV/u.

$-\frac{dN(t)}{dt} = \lambda N(t)$ (rate of nuclear decay); $N(t) = N_0 e^{-\lambda t}$ (solution to decay equation);

$T_{1/2} = \frac{\ln 2}{\lambda}$ (half-life for radioactive decay); $T_{\mathrm{mean}} = \frac{1}{\lambda}$ (mean lifetime for radioactive decay);

Outcome 2: Meaning of equations and manipulation for quantitative relationships

$E_l = l(l+1)\frac{\hbar^2}{2I}$ (Calculate transition energies, photon wavelengths for emission or absorption—differences in energy levels subject to quantum transition rule $\Delta l = \pm 1$);

$E_n = (n + \frac{1}{2})\hbar\omega = (n + \frac{1}{2})\hbar\sqrt{\frac{k'}{\mu}}$ (Calculate transition energies, photon wavelengths for emission or absorption—differences in energy levels subject to quantum transition rule $\Delta n = \pm 1$);

Note that both n and l can change during a transition in the above two equations.

$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$ (Use to determine probability of a state of energy E being occupied—same as fraction occupied. Find Fermi energy if other quantities are known, etc.);

$I = I_S(e^{eV/kT} - 1)$ (Use to calculate current for given voltage applied, or temperature dependence of current, etc.)

$N(t) = N_0 e^{-\lambda t}$ (Determine half-life given decay rate, or amount of material of a given radioactive species either at some time, or initially)

$E_B = (ZM_H + Nm_n - {}^{A}_{Z}M)c^2$ (Use to find binding energy, or amount of energy available for decay, or amount of energy liberated during a decay—comparing original binding energy with those of daughter products, etc.)

Outcome 3: Apply equations to physical systems with boundary conditions
N/A

Outcome 4:
Analyze energy-level diagrams for diatomic molecule rotation and vibration, and allowed transitions. Analyze *p*-*n* junction current diagram. Analyze Segré diagram for nuclear decay. Interpret Fermi-energy diagram for qualitative discussion of occupation of available states. Interpret binding energy curve.

Outcome 5:
Be able to evaluate the precision of results. Know and appreciate the approximations (e.g. liquid-drop model of nucleus)